

## Comparison of Control Performance for a Low-cost DC Motor with Single-loop and Cascade Control Architectures

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**Abstract:** A brushed DC motor is an important machine and widely used in the industry and in many of today's mechatronics systems such as mobile robots, robot arms, and other industrial applications because of its simplicity, ease of control, and reasonable cost. The position control of a DC motor is crucial for a precision control system and it is well known that the mathematical model is very crucial for a control system design. For a DC motor, there are many models and control architectures to achieve a good performance; (accuracy, and robustness according to its application). The aim of this paper is to investigate the performance comparison of a position control for a low-cost DC motor with simple position feedback and cascade control architectures. A low-cost DC motor is modeled and considered as a second-order system that involves lump parameters due to the absence of motor specification. In addition, the dynamic compensation is also included in the control model. The position control has accomplished in two types of control architecture, namely a single loop with a PD controller and a cascade control architecture composed of two loops, the velocity inner loop and the position outer loop with P controllers for both. MATLAB/Simulink model is used for modeling, simulation, and control of DC motor position, and then the control methods are deployed for real experiment assessment. Through analyzing and comparing, the result showed that both control methods achieve a good result of position reference tracking with no overshoot during a simulation time. More importantly, the cascade control methods clearly showed the improvement of achieving more accurate position control with the steady state error of two degrees compare to single loop control of 5 degrees in real hardware experiments tested.

**Keywords:** Cascade control; Single-loop control; DC Motor; Dynamic Compensation; PID controller

### 1. INTRODUCTION

DC motors are an important machine and widely used actuator in most control systems, it has uses in a variety of modern mechatronic systems, including robots, precise positioning devices, and industrial uses. DC motors positioning control is in particular popular and suitable for use in a situation when there is a need for precise and accurate response. For example, a pick-and-place application such as a robot arm requires precise position control to pick up required parts and place them in the correct position. Another, example for the production of Printed Circuit Boards (PCB), components must be placed precisely on the board before the soldering process [1].

We already know the mathematical model of the DC motor is very crucial for a control system design, there are many models

to represent the machine's behavior. However, the parameters of the model are also important because the mathematical model cannot provide correct behavior without the correct parameters in the model. In the implementation work, the mechanical and electrical of a low-cost DC motor are modeled as a second-order system that includes lump parameters. Those lump parameters are then estimated by using the Extended Kalman Filter.

The objective of this paper is to make a comparison of position control for low-cost DC motors by using two control methods the single loop and the cascade architecture. Generally, a simple single-loop control is widely used with the proportional and derivative control law to achieve a position control while cascade architecture involves the use of two control loops which the first control loop provides the set point for the second control loop. In a proposed architecture of cascade control loop

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arrangement, there are two controllers, the proportional controller is used for both the velocity inner loop and the position outer loop [2-4]. Both architectures are modeled and simulated under different position profiles by using MATLAB/Simulink environment. Then employed with hardware setup for real experiments to demonstrate and compared the performance of the system.

## 2. METHODOLOGY

### 2.1 Mathematical Model of DC Motor

An electric DC motor (in this case, a permanent magnet) provides the majority of the driving force for many applications. Mathematical modeling must be completed in order to produce the relationships between the current, voltage, and rotational speed that is necessary for the DC motor to be used in simulations. DC motor is composed of two parts the electrical part and the mechanical part are shown in the following figure:

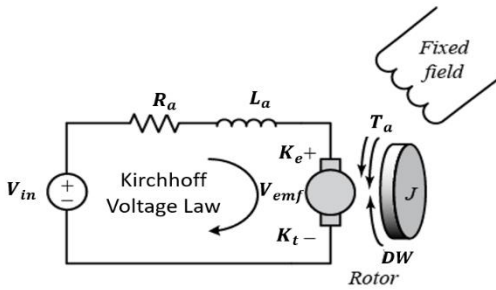


Fig. 1. DC motor equivalent circuit

The electrical part is obtained by applying Kirchoff voltage law to the circuit loop in Fig.1

$$V_{in} - V_{Ra} - V_{La} - V_{emf} = 0 \quad (\text{Eq. 2.1})$$

Where:

- $V_{in}$  = is an input voltage from the power source
- $V_{Ra}$  = is a voltage over the armature resistance
- $V_{La}$  = is a voltage drop over the armature inductance
- $V_{emf}$  = is a voltage induced by the coil

Substituting the first equation with  $V_{Ra} = R_a i_a$ ,  $V_{La} = L_a \frac{di_a}{dt}$ ,  $V_{emf} = K_e W$  yields following differential equation

$$V_{in} = R_a i_a + L_a \frac{di_a}{dt} + K_e W \quad (\text{Eq. 2.2})$$

Where:

- $W$  = is angular velocity of motor (rad/s)
- $R_a$  = is resistance ( $\Omega$ )
- $i_a$  = is current (A)
- $L_a$  = is the inductance (H)
- $K_e$  = is back electromotive force coefficient (v)

The mechanical part of the motor can be modeled by torque balance (energy balance) in the system the mechanical equation can be sated.

$$T_a = T_f + J\dot{W} \quad (\text{Eq. 2.3})$$

Where:

- $T_a$  = is rotor torque (Nm)
- $T_f$  = is torque of coulomb friction and viscous friction
- $J$  = is moment of inertia ( $Kgm^2$ )

Since  $T_f = T_c \text{sign}(W) + DW$  and  $T_a = K_t i_a$  substitute into equation 2.3 we get:

$$K_t i_a = T_c \text{sign}(W) + DW + J\dot{W} \quad (\text{Eq. 2.4})$$

Where:

- $K_t$  = is motor torque
- $T_c$  = is coulomb friction torque (Nm)
- $D$  = is coefficient viscous friction ( $Nm/rads^{-1}$ )

In practice, the inductance of the armature coil is very small and assumed approximately to zero ( $L_a \approx 0$ ).

From equation 2.2 and equation 2.4 finally, we obtain the dynamic of the motor sated as follows:

$$\dot{W} = -\left(\frac{K_e K_t + DR_a}{R_a J}\right)W + \frac{K_t}{R_a J}V_{in} - \frac{T_c}{J}\text{sign}(W) \quad (\text{Eq. 2.5})$$

According to equation 2.5 we get lumped parameter as

$$a = \left(\frac{K_e K_t + DR_a}{R_a J}\right)$$

$$b = \frac{K_t}{R_a J}$$

$$c = \frac{T_c}{J}\text{sign}(W)$$

Then equation 2.5 will be simplified as:

$$\dot{W} = -aW + bV_{in} - c\text{sign}(W) \quad (\text{Eq. 2.6})$$

### 2.2 Parameters identification

Since the low-cost DC motor do not have parameters, we then estimate the parameter by using Kalman filter algorithm. We need to write a system presented in matrix form and let  $x_1 = W, x_2 = a, x_3 = b, x_4 = c$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_2x_1 + x_3u - x_4 \text{sign}(W) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{Eq. 2.7})$$

The state model and the measurement model for the EKF algorithm are defined as:

$$\begin{aligned} x_k &= f_d(x_{k-1}, u_{k-1}) + v_{k-1} \\ y_k &= h_d(x_k, u_k) + w_k \end{aligned}$$

Where:

- $x_k$  = state matrix
- $y_k$  = measurement matrix
- $k$  = time step
- $u$  = input control matrix

$w$  and  $v$  are independent Gaussian white noise distributions with the covariance  $Q$  and  $R$  respectively. The EKF algorithm is divided into two steps: the time update and the measurement update [5].

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**Time Update:**

$$\begin{aligned} \hat{x}_{k|k-1} &= f_d(\hat{x}_{k-1|k-1}, u_{k-1}) && \text{state estimate} \\ P_{k|k-1} &= A_{k-1}P_{k-1|k-1}A_{k-1}^T + Q_{k-1} && \text{error covariance} \end{aligned}$$

**Measurement Update:**

$$\begin{aligned} \hat{y}_{k|k-1} &= h_d(\hat{x}_{k|k-1}, u_k) \\ P_{xy,k|k-1} &= P_{k|k-1}C_k^T \\ P_{yy,k|k-1} &= C_kP_{k|k-1}C_k^T + R \\ W_k &= P_{xy,k|k-1}P_{yy,k|k-1}^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + W_k(y_k - \hat{y}_{k|k-1}) \\ P_{k|k} &= P_{k|k-1} - W_kP_{yy,k|k-1}^{-1}W_k^T \end{aligned}$$

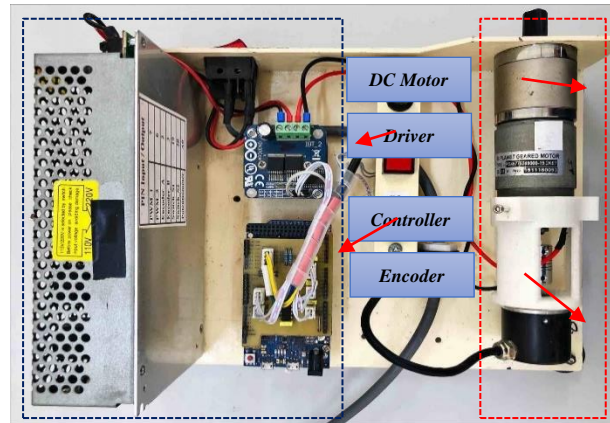
We obtain the Jacobian matrix by applying Taylor series expansion to linearize the nonlinear functions  $f_d$  and  $h_d$ . The matrix  $A_k$  is the Jacobian matrix of the nonlinear state function where:

$T_s$  = sampling time

**2.3 Hardware Implementations**

This section explains the components for experiment used including main controller, dc motor, dc motor driver, and incremental encoder. In Fig. 2. show the laboratory testbed that used to implement in this work.

Arduino due are used as the main controller. The BTS6970 module H-bridge motor driver, with overheating and overcurrent protection is employed, together with an incremental encoder sensor are used as the position feedback with 1000 pulse per revolution. Especially, the dc motor used in this paper is a low-cost dc motor that the manufacture did not provide the motor’s parameters, and this problem is challenging for control system. First of all, we need to identifies the parameters of the motor, to do that we use the estimation method which is an extended Kalman filter to estimate the lumped parameters as showed in the subsection 2.2 and Eq. 2.6. accordingly.



**Fig. 2.** Laboratory testbed: DC motor with a rotary encoder on the right and microcontroller (Arduino Due), motor driver (BTS7960) and power supply on the left.

**Table 1.** Parameters estimation

Description	Estimate state	Value
Lump parameter $a$	$a$	26.85
Lump parameter $b$	$b$	85.68
Lump parameter $c$	$c$	31.85

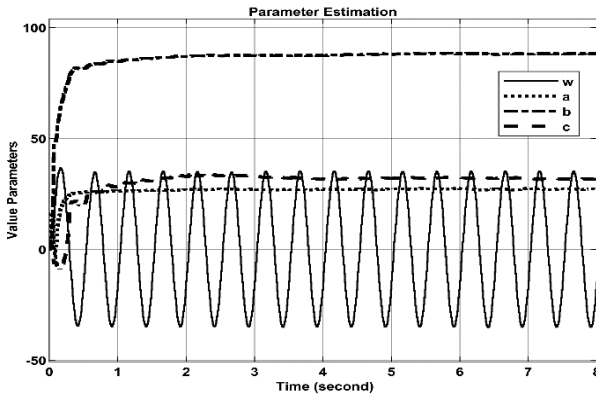


Fig. 3. Lump parameters estimation for low-cost dc motor

2.4 The System Block Description

In the following Fig. 4., the position control of the DC motor with a single loop control architecture that consists of proportional and derivative control law. The motor plant is derived as the first order of differential equation and in order to get the position feedback we need to take the integral of angular acceleration and angular velocity accordingly. To obtained the position error we have to take the position feedback (from encoder sensor for real experiment) minus with the desire position. This position error then went through the PD controller to generate the desired voltage for the motor plant. The compensator also included to improve the performance of the system meaning that to compensate the position error converge to zero over time. Last but not least, the model of the dc motor has included the friction that make the system more reliable.

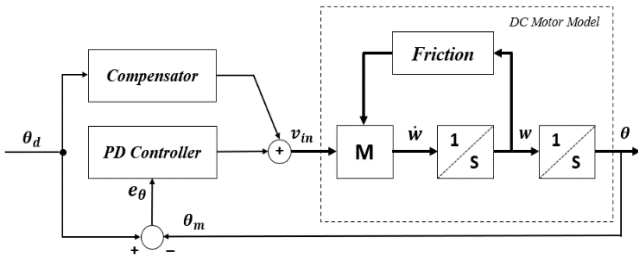


Fig. 4. Position control with single loop architecture

Another method is this paper is proposed alternative approve of position control by using cascade control architecture as illustrated in the Fig. 5. Instead of using a single feedback loop, this control strategy uses two dependent loops, the velocity inner loop and the position outer loop.

To obtained the velocity inner loop and position outer loop we have to take an integral of angular acceleration and integral of angular velocity from the motor plant. The position then feedback to system by minus with the desire position to get the

position error. This position error went through the first proportional controller and minus with the desire inner velocity loop then we got the velocity error of the system. Then the error of velocity went through the second proportional controller to get the desire input to the motor plant. In addition, the system also included the compensator like a single loop architecture to improve the system performance, as well as the motor also include friction to make system more reliable compare the real DC motor.

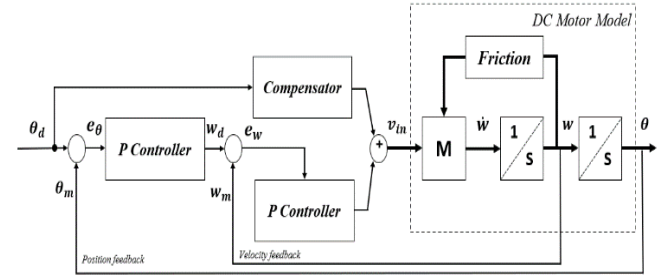


Fig. 5. Position control with cascade control architecture

2.5 Controller Design

To obtained controller gain according to the Fig. 4. And Fig. 5. The governing equation of the position control with single loop architecture can be written in term of the position error as:

$$e_{\theta} + (bK_D + a)e_{\dot{\theta}} + bK_p e_{\theta} = \ddot{\theta}_d + a\dot{\theta}_d \quad (\text{Eq. 2.8})$$

Also, the equation of the position control with cascade control loop architecture can be stated in term of position error as the following:

$$e_{\theta} + (a + bK_2)e_{\dot{\theta}} + bK_2K_1 e_{\theta} = \ddot{\theta}_d + (a + bK_2)\dot{\theta}_d \quad (\text{Eq. 2.9})$$

In order to find controller gain of these two methods we can compared the Eq. 2.8 and Eq. 2.9 with the standard form of second order differential equation as the following:

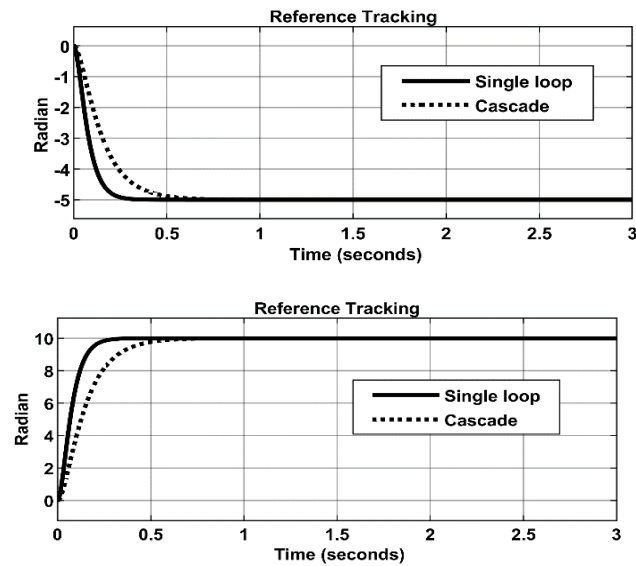
$$\ddot{X} + 2\zeta w_n \dot{X} + w_n^2 X = 0 \quad (\text{Eq. 2.10})$$

From the Eq. 2.8 to Eq. 2.10, we get the controller gain as following by choosing  $\zeta = 1$ , and  $w_n = 2\pi 4$ .

**Table 2.** Controller gain for single loop and cascade control

Controller	Value
$K_p$	$= \frac{w_n^2}{b} = 7.37$
$K_D$	$= \frac{2\zeta w_n - a}{b} = 0.27$
$K_1$	$= w_n^2 / 2\zeta w_n - a = 26.97$
$K_2$	$= \frac{2\zeta w_n - a}{b} = 0.27$
Single loop Compensator	$= \ddot{\theta}_d + a\dot{\theta}_d$
Cascade loop Compensator	$= \ddot{\theta}_d + (a + bK_2)\dot{\theta}_d$

**3. RESULTS AND DISCUSSION**

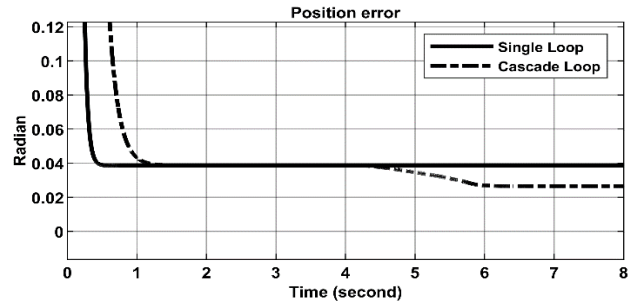


**Fig. 6.** Simulation result of position control case 2

Otherwise, compare to single loop, cascade control has slower rise time and reach the desire target about 0.7 in 2 scenarios with  $K_1 = 26.97$  and  $K_2 = 0.27$ . In the other hands, both control methods have achieved the desired target and there is no overshoot during the simulation time.

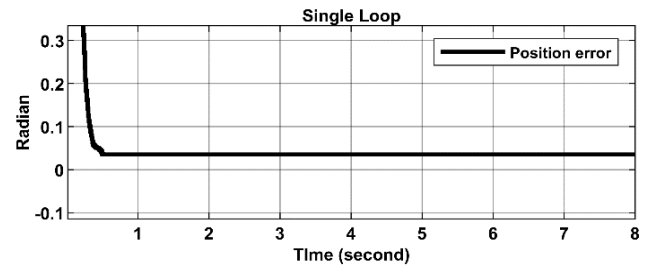
Moreover, in position control, the position error is the most important factor to indicate the accuracy for the system response. In Fig. 7. shown the comparison of position error of both methods, the single loop method has the position error of about

0.04 radians whereas the cascade loop has the position error of about 0.025 radians. It clearly indicated that the proposed cascade architecture has achieved better accuracy than a simple loop control about 0.015 radian (equal to 0.85 degree) during simulation time of 8 second.

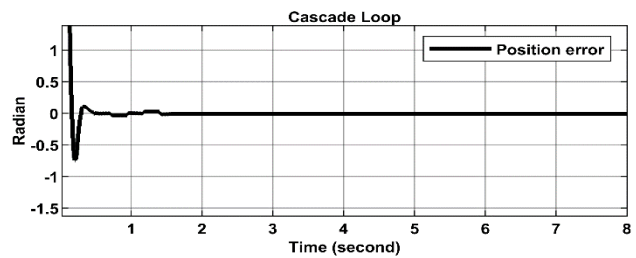


**Fig. 7.** Simulation result of position error

For reals experiment result we can see that in Fig. 8. and Fig. 9. of the position error, the single loop control has about 0.095 radians of position error which is equal to 5.44 degrees compare to cascade loop about 0.035 radian that equal to 2 degrees. Therefore, the performance of cascade architecture has better performance with the accuracy of 2 degrees compare to single loop control of 5 degree during the conduct experiment result.



**Fig. 8.** Hardware result of position error single loop control



**Fig. 9.** Hardware result of position error cascade loop control

**4. CONCLUSION**

An overview of different DC motor control approaches is given in this work. PID controller is employed for both architectures. By comparing simulation results, it is concluded that positioning control via cascade control loop and single

control loop they both reach the desire target under different position profiles. A single loop in all two conditions have a faster rise time than cascade control loop and they both have no overshoot in overall. Otherwise, from both simulation and experiment result, it clearly shown that the performance of cascade control loop has achieve more accurate of position error about 0.085 degrees and 2 degrees of simulation and hardware result accordingly. The cascade control having overshoot with real experiment but after a second it converges to zero and obtained a good accuracy compare to single loop control.

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In the simulation result of single loop and cascade control loop with conventional PID under different position profiles, in Fig. 5. and Fig. 6. point out a good result of reference position tracking of both methods. The single loop control has achieved the desire target with the rise time about 0.3 seconds in 2 conditions corresponding of  $K_p = 7.37$  and  $K_D = 0.27$

## REFERENCES

- [1] Bishop, R. H. (2002). The mechatronics handbook-2 volume set. CRC press.
- [2] Komić, A., & Dubravić, A. (2020, March). DC motor dynamics data acquisition, parameters estimation and implementation of cascade control. In 2020 19th International Symposium INFOTEH-JAHORINA (INFOTEH) (pp. 1-5). IEEE.
- [3] Bacac, N. & Slukic, V. & Puskaric, Miroslav & Stih, B. & Kamenar, Ervin & Zelenika, Sasa. (2014). Comparison of different DC motor positioning control algorithms. 1654-1659. 10.1109/MIPRO.2014.6859832.
- [4] Soares, D. M., Calil, H. A., & Stephan, R. M. (2019, December). Cascade control vs full-state feedback. In 2019 IEEE 15th Brazilian Power Electronics Conference and 5th IEEE Southern Power Electronics Conference (COBEP/SPEC) (pp. 1-6). IEEE.
- [5] Yean, S., Peou, T., Sethy, B., & Srang, S. (2021, December). PD Controller and Dynamic Compensation Design for a DC Motor based on Estimated Parameters. In 2021 International Conference on Advanced Mechatronics, Intelligent Manufacture and Industrial Automation (ICAMIMIA) (pp. 7-12). IEEE.